DETERMINATION OF THE EFFECTIVE THERMAL CONDUCTIVITY OF A PLATE IN NONSTEADY EXPERIMENT

E. A. Belov, G. Ya. Sokolov, and A. S. Starkov

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The article describes a method of finding the effective thermal conductivity of multilayered structures at the nonsteady stage of the experiment and an experimental installation with automatic systems for setting the experimental regime and processing the experimental data.

In refrigeration and cryogenic engineering inhomogeneous heat-insulating structures are ever more widely used; they consist of alternating layers of heat and moisture insulation and coatings ensuring the indispensable mechanical strength. In dependence on the production technology, the thermal conductivity of such systems is described either by a step function or by a continuous function of the coordinate. In the general case, the effective thermal conductivity of such an inhomogeneous medium bounded by plane surfaces is determined by the relation

$$\lambda_{\text{ef}} = \delta \left(\int_0^\delta \frac{dx}{\lambda(x)} \right)^{-1}. \tag{1}$$

So far the only method of the experimental determination of the effective thermal conductivity of an inhomogeneous medium is the method of steady thermal regime which requires a considerable length of time. On the other hand, the thermal conductivity of homogeneous materials is often measured by the unsteady method of bicalorimeter [1]. An important advantage of this last method is that the heat insulation of metallic structures can be investigated directly on the object, without special specimens having to be made [2]. Below we show that it is possible to use the thermal regime of the bicalorimeter for rapid measurements of the effective thermal conductivity of inhomogeneous media.

The thermal diagram of the method is shown in Fig. 1. On one side the investigated material in the form of a plate is in contact with the metallic base whose temperature is maintained unchanged in the course of the entire experiment. On the opposite side a previously overheated heat-conducting disk is mounted. The overheating of the disk is chosen within the limits 10-15°K, and here the thermophysical characteristics may be regarded as not dependent on the temperature. Under certain conditions, which will be specified below, the temperature field in the region of the material adjacent to the disk is close to unidimensional. The thermal perturbation $t(x, \tau)$ induced in the investigated body can be found from the solution of the boundary-value problem for the equation of thermal conductivity:

$$\frac{\partial}{\partial x} \left(\lambda(x) \frac{\partial t}{\partial x} \right) - c(x) \rho(x) \frac{\partial t}{\partial \tau} = 0 \tag{2}$$

with zero initial condition (without loss of generality we adopt the unperturbed temperature of the material as the reference point)

$$t(x, 0) = 0 \tag{3}$$

and the boundary conditions

$$-\lambda(\delta) \frac{\partial t(\delta, \tau)}{\partial x} = C_{\text{d}} \frac{\partial t(\delta, \tau)}{\partial \tau}, \tag{4}$$

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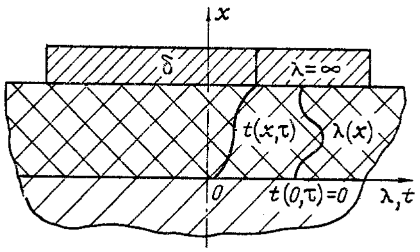

Fig. 1

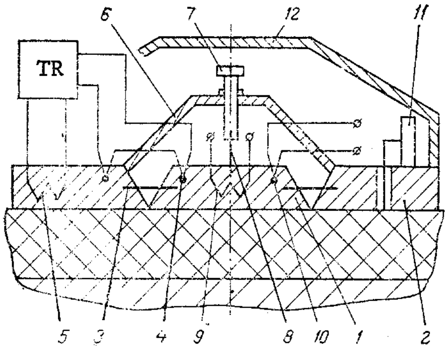

Fig. 2

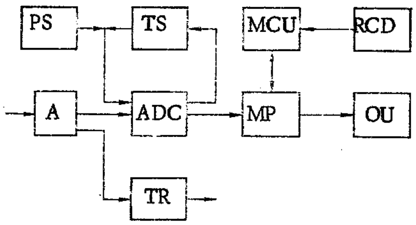

Fig. 3

Fig. 1. Thermal diagram of the method.

Fig. 2. Design of the thermal probe: 1) copper core; 2) protective ring; 3) pipes for fastening the core; 4) differential thermal pickup; 5) heater; 6) shell; 7) micrometric screw; 8) needle; 9) heater; 10) thermal pickup; 11) limit switch; 12) jacket.

Fig. 3. Functional diagram of the calculating and control unit.

$$t(0,\ \tau)=0. \tag{5}$$

We bring into consideration the averaged temperature (weighted instants) according to the rule [3]:

$$\tilde{t}(x,\ k)=\int_0^\infty t(x,\ \tau)\tau^k d\tau, \tag{6}$$

where k = 0, 1, 2, ... is an integer.

When we multiply relation (2) by τ^k and integrate with respect to time within the limits 0, ∞, we obtain an equation for finding the function $\tilde{t}(x,\ k)$:

$$\frac{\partial}{\partial x}\left(\lambda(x)\frac{\partial\tilde{t}(x,\ k)}{\partial x}\right)=-kc(x)\rho(x)\tilde{t}(x,\ k-1). \tag{7}$$

The boundary condition (4) is transformed to

$$-\lambda(\delta)\frac{\partial\tilde{t}(\delta,\ 0)}{\partial x}=C_d t_0 \tag{8}$$

for k = 0 and to

$$\lambda(\delta)\frac{\partial\tilde{t}(\delta,\ k)}{\partial x}=C_d kt(\delta,\ k-1) \tag{9}$$

for k = 1, 2, ...

In relation (8) t_0 denotes the initial overheating of the disk in relation to the unperturbed temperature of the surface of the investigated material. In particular, with k = 0 successive integration of Eq. (7) leads to the relation

$$\tilde{t}(\delta,\ 0) = C_d t_0 \int_0^\delta \frac{dx}{\lambda(x)} = C_d t_0 \delta \lambda_{ef}^{-1}. \quad (10)$$

The fundamental difference between the obtained expression for λ_{ef} and known expression for the steady regime consists in the fact that the integral $\tilde{t}(\delta,\ 0)$ can already be found at the regular stage of the thermal process where the temperature of the disk $t(\delta,\ \tau)$ is described with the required accuracy by the exponential law

$$\tilde{t}(\delta,\ 0) = \int_0^{\tau_1} t(\delta,\ \tau)\, d\tau + \frac{t(\delta,\ \tau_1)}{m}. \quad (11)$$

From the full thermal resistance of the investigated material we can separate the specific contact thermal resistance P_c which has to be taken into account as correction. The final formula for calculating thermal conductivity has the form

$$\lambda_{ef} = \frac{C_d t_0 \delta}{\int_0^{\tau_1} t(\delta,\ \tau)\, d\tau + \frac{t(\delta,\ \tau_1)}{m}} \left(1 + \frac{P_c}{P_r}\right), \quad (12)$$

The described method was realized in an automatic installation consisting of a remote thermal probe and a computing and control unit. The main design elements of the thermal probe (see Fig. 2) are the copper core 1 made in the form of a truncated cone with 30 mm diameter of the contact base, and the protective ring 2 with 140 mm outer diameter. The core is rigidly mounted with the aid of three thin-walled nickel pipes 3 at the center of the protective ring with a guaranteed clearance of 0.5 mm. Maintenance of zero temperature difference between the core and the protective ring is ensured with the aid of the electronic thermoregulator (TR), the differential thermal pickup 4, and the wire heater 5 placed in radial grooves of the protective ring. This makes it possible with the indispensable degree of approximation to fulfill the condition that the temperature field in the central zone of action of the thermal probe be unidimensional.

In the realized installation the ratio of the radii of the protective ring and of the core was chosen to be 4.7, and then the relative error of measurement due to the nonunidimensionality of the field does not exceed 1% for specimens with thickness $\delta \leq 30$ mm [2]. The outer surface of the core is protected by the adiabatic shell 6 which prevents heat flow from the core to the environment. Thanks to the wide area of contact between the shell and the protective ring and the use of a highly heat conducting lubricant based on BeO we managed to ensure adiabatic conditions of the core and unidimensionality of the temperature field in the material with the aid of one TR. Matching of the contact faces of the core and of the protective ring is effected by the micrometric screw 7 with needle 8. For the preliminary overheating of the thermal probe at the preparatory stage of the experiment and for the execution of temperature measurements at the working stage, the wire heater 9 and the measuring thermal pickup 10 are mounted in the body of the core. On the contact base of the protective ring the mechanical sensor 11 is mounted, and this forms an electric pulse at the instant when the thermal pickup is placed on the surface of the investigated material. The entire structure is covered by the decorative jacket 12, and the free space is filled with plastic foam pellets.

The computing and control unit is based on a microprocessor set of the series K 584. It is intended for ensuring the maintenance of the stipulated regimes of the thermal pickup at the preparatory and working stages of the experiment, and also the calculation and recording of the results of measurements on a digital display. The functional diagram of the unit is shown in Fig. 3.

The signals transmitted from the thermal pickup are amplified in the unit of precision amplifiers (A) and converted to digital code with the aid of the 12-bit analog-to-digital converter (ADC) of integration type.

All the logic and arithmetical operations are carried out by the microprocessor (MP) whose rigid working algorithm is stored in the memory of the microprogram control unit (MCU) The temporary store (TS) and the permanent store (PS), made with the use of TTL logic of the series K 155, ensure the storage of the necessary current data, and also of the calculation and auxiliary constants. The operating regime of the unit is chosen with the aid of the logic regime control device (RCD). The data processed by the MP are transmitted to the output unit (OU) which is a binary-decimal data register and a matching and coding device for linking with the luminescent indicator display.

Maintenance of the specified thermal regimes of the thermal probe is ensured by an electronic three-parameter ShIM-regulator which has independent setting of all three (PID) control parameters. This makes it possible, with the use of the method [4], to attain optimal setting of the regulator without iterations. At the preparatory stage of the experiment lasting about three minutes the thermal probe is heated, and then it is placed on the free surface of the material to be checked. The duration of the working stage of the experiment depends on the geometric and thermophysical characteristics of the material, and for effective heat-protective coatings with thickness up to 30 mm it does not exceed 25 min. During the experiment the computing and control unit processes the data transmitted by the thermal probe, and after the regular stage of the thermal process has been reached, the value of λ_{ef} of the investigated material is shown on the digital display.

A series of control experiments was carried out with two-layered and three-layered specimens made in the form of plates, 250 × 250 × 30 mm^3 in size, of materials with known thermophysical characteristics. These materials were foam plastic PS1-100, polymethyl metacrylate PMMA, and plastic ST in various combinations. The maximal deviation of the experimental data from the calculated ones, carried out for a flat fabricated wall, does not exceed 4.5%.

NOTATION

δ, thickness of the layer of heat insulation; x, coordinate; $t(x, \tau)$, temperature at the point of the material with the coordinate x at the instant τ; λ, thermal conductivity; λ_{ef}, effective thermal conductivity; c, heat capacity; ρ, density of the material; C_d, heat capacity of the disk per unit area of the base; m, cooling rate of the disk; P_c, thermal contact resistance; P_r, thermal resistance of the investigated layer.

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